Spectral Methods for Theoretical Computer Science

Shravas Rao, Portland State Computer Science

Query Complexity

Query Complexity

- We can simplify the problem of understanding the relative power of randomness and quantum computing by only considering *queries* to the input

Query Complexity

- We can simplify the problem of understanding the relative power of randomness and quantum computing by only considering *queries* to the input
- Given a function $f : \{0,1\}^n \rightarrow \{0,1\}$, consider the largest eigenvalue of an associated matrix - spectral sensitivity

Query Complexity

- We can simplify the problem of understanding the relative power of randomness and quantum computing by only considering *queries* to the input

- Given a function $f : \{0,1\}^n \to \{0,1\}$, consider the largest eigenvalue of an associated matrix - spectral sensitivity

- $\mathrm{D}(f) = O(\mathrm{Q}(f)^4)$ for total functions - Aaronson Ben-David Kothari R Tal 2021

Query Complexity

- We can simplify the problem of understanding the relative power of randomness and quantum computing by only considering *queries* to the input
- Given a function $f : \{0,1\}^n \to \{0,1\}$, consider the largest eigenvalue of an associated matrix - spectral sensitivity
- $\mathrm{D}(f) = O(\mathrm{Q}(f)^4)$ for total functions - Aaronson Ben-David Kothari R Tal 2021
- What are other appliations of spectral sensitivity? CRII: AF: Applications of Spectral Sensitivity to Query and Communication Complexity

Graph Sparsification

Graph Sparsification

- Given a graph, can we construct a “similar” graph with fewer edges?

Graph Sparsification

- Given a graph, can we construct a “similar” graph with fewer edges?
- “Similar” - set of eigenvalues is the same up to small perturbations, implications for mixing time of random walks

Graph Sparsification

- Given a graph, can we construct a “similar” graph with fewer edges?
- “Similar” - set of eigenvalues is the same up to small perturbations, implications for mixing time of random walks
- Alev-R 2024 - for certain graphs, this is possible (using expander graphs)

Graph Sparsification

- Given a graph, can we construct a “similar” graph with fewer edges?
- “Similar” - set of eigenvalues is the same up to small perturbations, implications for mixing time of random walks
- Alev-R 2024 - for certain graphs, this is possible (using expander graphs)
- Application - algorithms for counting the number of colorings in graphs

Fourier Transform of the Slice

Fourier Transform of the Slice

- Slice - graph with vertex set subsets of {1, 2, …, n} of size k, edge between two vertices if they differ in one element

Fourier Transform of the Slice

- Slice - graph with vertex set subsets of {1, 2, …, n} of size k, edge between two vertices if they differ in one element
- Given a vector, can compute change of basis of eigenvectors of adjacency matrix quickly - R 2022+

Fourier Transform of the Slice

- Slice - graph with vertex set subsets of {1, 2, …, n} of size k, edge between two vertices if they differ in one element
- Given a vector, can compute change of basis of eigenvectors of adjacency matrix quickly - R 2022+
- Application, given set system $\mathcal{S} \subseteq \binom{[n]}{n/4}$, can detect if there are two disjoint sets in time $O\left(\binom{n}{n/4}\right)$

Fourier Transform of the Slice

- Slice - graph with vertex set subsets of {1, 2, …, n} of size k, edge between two vertices if they differ in one element
- Given a vector, can compute change of basis of eigenvectors of adjacency matrix quickly - R 2022+
- Application, given set system $\mathscr{S} \subseteq \binom{[n]}{n/4}$, can detect if there are two disjoint sets in time $O\left(\binom{n}{n/4}\right)$
- Possibly use for subset sum?